Rolle's Theorem and the Mean Value Theorem Calculus of a Single Variable X. Du

- **Rolle's Theorem**: If a function $f(x)$ is continuous on [a, b] where $a < b$, differentiable on (a, b), and $f(a) = f(b)$, then there exists a $c \in (a, b)$ such that $f'(c) = 0$.
    - Using Rolle's Theorem:
        - First establish that $f(x)$ is continuous on [a, b] and differentiable on (a, b).
        - Establish that $f(a) = f(b)$.
        - State that Rolle's Theorem is applicable as those conditions are met.
        - Take the derivative of $f(x)$ and set that equal to 0.
        - Solve for x to find the value of c satisfied by Rolle's Theorem.
- **Mean Value Theorem**: If a function $f(x)$ is continuous on [a, b] where $a < b$ and differentiable on (a, b), then there exists a c where $c \in (a, b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.
    - Using the Mean Value Theorem:
        - First establish that $f(x)$ is continuous on [a, b] and differentiable on (a, b).
        - State that the Mean Value Theorem applies as those conditions are met.
        - Find the slope of the line connecting (a, $f(a)$) and (b, $f(b)$).
            - i.e. find $\dfrac{f(b) - f(a)}{b - a}$
        - Take the derivative of $f(x)$ and set that equal to the slope found previously.
        - Solve for x to find the value of c satisfied by the Mean Value Theorem.
    - Note similarities between Rolle's Theorem and the Mean Value Theorem.
        - Rolle's Theorem is a specific case of the Mean Value Theorem.